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INTRODUCTION OF GENERALIZED LAPLACE-FRACTIONAL MELLIN TRANSFORM

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ABSTRACT
In present era, Fractional Integral Transform plays an important role in various fields of mathematics and Technology. Mellin transform has an many application in navigations, correlaters, in area of statistics, probability and also solving in differential equation. Fractional Mellin transform is integral part of mathematical modeling method because of its scale invariance property.

The aim of this paper is to generalization of Laplace-Fractional Mellin Transform. Analyticity theorem for the Generalized Laplace-Fractional Mellin Transform is proved.

KEYWORDS: Laplace transform, Mellin transform, Fractional Mellin transform, Laplace-Fractional Mellin Transform, Generalized function.

INTRODUCTION
Laplace transformation belongs to a class of analysis method called integral transformation which is studied in the field of operational calculus. This method includes the Fourier Transform, Mellin Transform etc. All these transformations which are provide us alternative way to analyze the spectra of different signals [1].

In the time domain linear system can be given by a differential equation of integer or non integer order type, using Laplace transform, these systems are represented by transfer function corresponding to an implicit or explicit derivative transmittance. The most commonly used of Laplace transformation requires that the forcing function are said to "turn on" just after t=0 [2].

Mellin transform, a kind of nonlinear transformation is widely used in signal processing. Mellin transform is implemented as a fast Mellin Transform [3]. In literature, 2D Mellin transform implementation appears in conjunction with Fourier transform popular as Fourier–Mellin transform used to extract rotation scale and translation invariance. In the visual navigation, algorithms are usually known to be computationally heavy and time consuming, while the time-to-impact computation of the imaged object is straightforward by using the Mellin transform base correlators [4, 5].

Fractional Mellin transform is extension of Mellin transform. Fractional Mellin transform firstly introduced into the field of image encryption by NR Zhou etl [6]. For more image encryption algorithms based on the Fractional Mellin transform and its variants, one can refer to [7, 8]. This transform becomes a new way used in visual navigation since it can control the range of rotation and scaling.
Motivated by the extension of Laplace and Mellin transform, which have been recently considered by many authors. In this paper we aim to introduce new transform Laplace-Fractional Mellin transform. It will be more applicable in the different area of science and technology.

In the present paper we have defined distributional Laplace-Fractional Mellin transform. Analyticity theorem of Laplace-Fractional Mellin transform is proved. Notation and terminology as per zemanion [15].

LAPLACE-FRACTIONAL MELLIN TRANSFORM

Definition

The Laplace-fractional Mellin transform with parameter $\boldsymbol{\theta}$ of $\mathbf{f(x,y)}$ denoted by $\mathbf{LFrMT\{f(t,x)\}}$ performs a linear operation, given by the integral transform

$$\mathbf{LFrMT\{f(t,x)\} = F_\theta\{f(t,x)\}(s,u) = F_\theta(s,u) = \int_0^\infty \int_0^\infty f(t,x) K_\theta(t,s,x,u)\, dt\, dx,} \quad \dots\dots\dots(2.1)$$

where,

$$\mathbf{K_\theta(t,s,x,u) = x^{\frac{2\pi i u}{\sin\theta}} e^{\frac{\pi i}{\tan\theta}(u^2+\log^2 x)-st}} \qquad \mathbf{0<\theta \le \frac{\pi}{2}} \quad \dots\dots\dots(2.2)$$

Test function space $\mathbf{LM^{\alpha,\beta}_{r,b}}$

An infinitely differentiable complex valued smooth function $\boldsymbol{\varphi}$ **on** $\mathbf{R^n}$ belongs to $\mathbf{E(R^n)}$, if for each compact set $\mathbf{I \subset S_a}$, where $\mathbf{S_a = \{x : x \in R^n, |x| \le a, a > 0\}, I \in R^n}$. And $\mathbf{k}$ be the open sets in $\mathbf{R_+ \times R_+}$ such that

$$\boldsymbol{\gamma}_{\mathbf{E,b,l,q}}(\boldsymbol{\varphi}) = \sup_{\substack{\mathbf{t \in k} \\ \mathbf{x \in I}}} \left|\mathbf{e^{bt} D_t^l D_x^q\, \varphi(t,x)}\right| < \infty \;, \qquad \boldsymbol{l, q = 0,1,2\dots.} \quad \dots\dots\dots(2.3)$$

The space $\mathbf{LM^{\alpha,\beta}_{r,b}}$ are equipped with their natural Hausdoff locally convex topology $\boldsymbol{\mathcal{T}}^{\alpha,\beta}_{\mathbf{r,b}}$. This topology is respectively generates by the total families of seminorms $\{\boldsymbol{\gamma}_{\mathbf{E,b,l,q}}\}$ given by (2.3).

DISTRIBUTIONAL GENERALIZED LAPLACE-FRACTIONAL-MELLIN TRANSFORM (LMRT)

Let $LM^{\alpha,\beta*}_{r,b}$ is the dual space of $LM^{\alpha,\beta}_{r,b}$. This space $LM^{\alpha,\beta*}_{r,b}$ consist of continuous linear functional on $LM^{\alpha,\beta}_{r,b}$. The distributional Laplace-Fractional Mellin transform of $f(t,x) \in E^*(R^n)$ is defined as

$$LFrMT\{f(t,x)\} = F_\theta\{f(t,x)\}(s,u)$$

$$= \langle f(t,x), K_\theta(t,s,x,u)\rangle \qquad \cdots\cdots\cdots(3.1)$$

where

$$K_\theta(t,s,x,u) = x^{\frac{2\pi i u}{\sin\theta}} e^{\frac{\pi i}{\tan\theta}(u^2+\log^2 x)-st} \qquad \dots\dots\dots(3.2)$$

For each fixed $t(0<t<\infty)$, $s>0$ and $0<\theta\le\frac{\pi}{2}$, the right hand side of (3.1) has sense as the application of $f(t,x) \in LM^{\alpha,\beta*}_{r,b}$ to $K_\theta(t,s,x,u) \in LM^{\alpha,\beta}_{r,b}$.

ANALYTICITY THEOREM OF LAPLACE- FRACTIONAL MELLIN TRANSFORM

Let $f \in E^*(R^n)$ and Laplace-Fractional Mellin Transform defined by(3.1). Then $F_\theta(t,x)$ is analytic on $C_n$if $supp\ f \subset S_a$

Where $S_a = \{x : x \in R^n | |x| \le a, a > 0\}$

Moreover, $F_\theta(t,x)$ is differentiable and

$$D_v F_\theta(t,s,x,v) = \langle f(t,x), \quad D_v K_\theta(t,x,s,v)\rangle$$

Proof: Let $v = v_1, v_2, \dots\dots\dots. v_j \dots\dots. v_n$

we first prove that $\frac{\partial}{\partial v_j} F_\theta(s,v) = \langle f(t,x), \frac{\partial}{\partial v_j} K_\theta(t,x,s,v)\rangle$

For fixed $v_j \ne 0$

Choose two concentric circle C and C' with centre at v_j and radius at $r$nadr' respectively.

Such that$0 < r < r_1 < |v_j|$.

Let Δv_j be a complex increment satisfying

$$0 < |\Delta v_j| < r$$

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Consider,

$$\frac{F_\theta(t,s,x,v_j+\Delta v_j)-F_\theta(t,s,x,v_j)}{\Delta v_j}-\langle f(t,x),\frac{\partial}{\partial v_j}K_\theta(t,x,s,v)\rangle=\langle f(t,x),\psi_{\Delta v_j}(t,x)\rangle$$

Where

$$\psi_{\Delta v_j}=\left[\frac{K_\theta(t,s,x,v_j+\Delta v_j)-K_\theta(t,s,x,v_j)}{\Delta S}\right]-\frac{\partial}{\partial v_j}K_\theta(t,x,s,v_j\ )$$

For any fixed $x\in R^n$ and any fixed integer $q=(q_1,q_2\ldots\ldots.q_n)$

$$D_x^qK_\theta(t,x,s,v)=D_x^q\left[e^{-st}.e^{\frac{\pi i}{\tan\theta}(\gamma^2+\log^2x)}.x^{\frac{2\pi i\,\gamma}{\sin\theta}}\right]$$

$$=e^{-st}D_x^q\left[e^{\frac{\pi i}{\tan\theta}(\gamma^2+\log^2x)}.x^{\frac{2\pi i\,\gamma}{\sin\theta}}\right]$$

$$=e^{-st}D_x^q\left[\sum_{r=0}^{q}\sum_{q=0}^{r}\binom{q}{r}P(V).r!\,.x^{-q+r}\left(\frac{2\pi i}{\tan\theta}\right)^{r-q}(\log x)^{r-q}.C_r(x)K_\theta(x,v)\right]$$

Where

$$C_r(x)=\frac{1}{(r-2b)!}\left(\frac{1-\log x}{2x\log x}\right)^r,\qquad P(V)\text{ is polynomial in }V.$$

Since for any fix for any fixed $x\in R^n$ and any fixed integer q and θ ranging from 0 to $\frac{\pi}{2}$, $D_x^qK_\theta(t,s,x,v)$ is analytic inside on C'.

We have by Cauchy integral formula

$$D_x^q\psi_{\Delta v_j}(t,x)=\frac{1}{2\,\pi i}e^{-st}\int K_\theta(t,s,x,v)\frac{1}{\Delta v_j}\left\{\left[\frac{1}{z-v_j-\Delta v_j}-\frac{1}{z-v_j}\right]-\frac{1}{(z-v_j)^2}\right\}dz$$

$$=\frac{1}{2\,\pi i}e^{-st}\int K_\theta(x\ \bar v)\frac{1}{\Delta v_j}\left\{\left[\frac{1}{z-v_j-\Delta v_j}-\frac{1}{z-v_j}\right]-\frac{1}{(z-v_j)^2}\right\}dz$$

$$=\frac{\Delta v_j}{2\,\pi i}e^{-st}\int_c\frac{P(x\ \bar v)}{(z-v_j-\Delta v_j)(z-v_j)^2}\,dz$$

Where $\bar v=v_1,v_2\ldots v_{j-1},v,v_{j+1}\ldots v_n$

$$\therefore D_t^lD_x^q\psi_{\Delta v_j}(t,x)=\frac{\Delta v_j}{2\,\pi i}(-1)^lS^le^{-st}\int_c\frac{P(x\ \bar v)}{(z-v_j-\Delta v_j)(z-v_j)^2}\,dz$$

$\therefore$ But for all $z\in C'$ and x restricted to a compact subset R^n.

Moreover $|z-v_j-\Delta v_j|>r_1-r>0$ and $|z-v_j|=r_1$.

Therefore we have

$$\left|D_t^lD_x^q\psi_{\Delta v_j}(t,x)\right|=\left|\frac{\Delta v_j}{2\,\pi i}(-1)^lS^le^{-st}\int_c\frac{P(x\ \bar v)}{(z-v_j-\Delta v_j)(z-v_j)^2}\,dz\right|$$

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$$\leq \frac{|\Delta v_j|}{2\pi} L \int_C \frac{Q_1}{(r_1 - r)(r_1)^2} |dz|$$

$$\leq \frac{|\Delta v_j| \, L \, Q_1}{(r_1 - r) r_1}$$

$$\leq \frac{|\Delta v_j| \, Q}{(r_1 - r) r_1} \qquad \because L \, Q_1 = Q$$

Thus as $|\Delta v_j| \to 0$ and $D_t^l D_x^q \psi_{\Delta v_j}$ tends to zero.

Uniformly on the compact subset of R^n
Therefore it follows that $\psi_{\Delta v_j}(t,x)$ converges in $E(R^n)$to zero
Since $f \in E^*$; we conclude that (1.1) tends to zero.
Therefore, $F_\theta(x,v)$ is differentiable with respect to v_j .
But this is true for all $j = 1,2 \ldots n$
Hence $F_\theta(s,v)$ is analytic on $C^n$and
$D_x F_\theta(s,v) = \langle f(t,x), D_x K_\theta(t,s,x,v) \rangle$
Hence proved

CONCLUSION
In the present work analytical structure of Generalized Laplace-Fractional Mellin transform is presented.

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